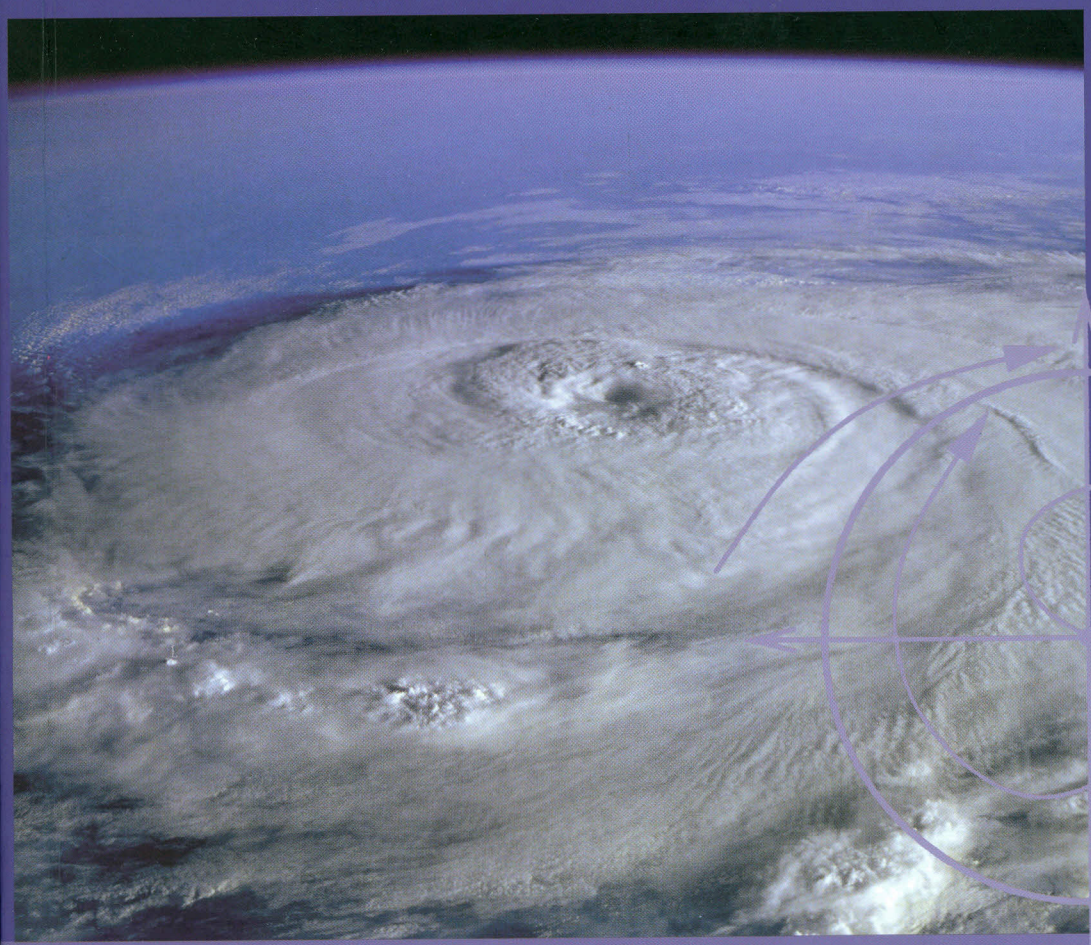
Dynamics of the Atmosphere

A Course in Theoretical Meteorology



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CAMBRIDGE

DYNAMICS OF THE ATMOSPHERE: A COURSE IN THEORETICAL METEOROLOGY

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